

TAXATION OF CAPITAL GAINS WITH DEFERRED REALIZATION

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ABSTRACT

This paper examines accepted methods of calculating the effect of deferred realization on the effective rate of capital gains tax paid by common shareholders. These methods are shown to be tailored to the special case of gains accrued in lump sum, unlike the usual scenario of gains accrued gradually over time. A widely-accepted method is also shown to overstate the effect of deferral by implicitly relying on the internal rate of return concept. The valuation-based method proposed here is designed for growth stocks, where gains are accrued gradually and realized in lump sum. The same method is used to calculate the firm's cost of retained earnings under a finite deferral period.

I. Introduction

THE economic consequences of deferred realization of capital gains accrued in common stocks concern financial theorists and macroeconomists alike. Financial theorists are concerned with the implications for the financing and investment policies of firms and individuals. By decreasing the effective rate of capital gains tax (CGT), deferred realization lowers the cost of financing the corporation by equity versus debt [19], and by retention versus stock issuance [28]. The decreased cost of retention increases the optimal scale of investment by established firms [28]. A decrease in the effective rate of CGT decreases the overall tax burden on corporate-source income, and more so the higher the firm's growth rate. This may have further effects on any tax-related investor clienteles [14]. For example, the uneven tax impact on growth and income stocks should increase the incentive of tax-exempt funds to hold growth stocks versus income stocks and bonds [32].

Macroeconomists are concerned with misallocation of resources. The realization-based taxation of capital gains in this and other countries is frequently blamed for inducing uneconomic behavior by investors. One manifestation of that behavior is the so-called "lock-in" effect on corporate shareholders, who may defer realization of accrued capital gains to reduce the effective tax rate on such gains [2, 3, 7, 11, 12, 13, 15, 17, 20, 31]. Auerbach [5] concludes that "this effect leads investors to accept a lower rate of return before tax than they would for new investments without accrued gains, resulting in a distorted allocation of capital and inefficient portfolio selection." Tendencies to defer realization of capital gains are likely to be augmented by the increased rate of CGT under the 1986 Tax Reform Act, and there is a renewed interest in the subject among scholars. Recent contributions by Shakow [27] and Auerbach [5] contain proposals for tax reforms designed to remove the incentive to defer realization of capital gains.

The partial-equilibrium analysis offered here reexamines the accepted methods of calculating the effect of deferred realization on the effective CGT rate and the cost of corporate retained earnings. Those methods are shown to be tailored to gains which accrue in a lump sum, unlike the typical scenario of gains accruing gradually over time. Given this limitation, the traditional method and that of King [16] are found to be correctly based on the principle of valuation, unlike the method of Bailey [6], Auerbach [4] and Protopapadakis [26], which is incorrectly based on the Internal Rate of Return concept. As a result, the latter method is shown to overstate significantly the effect of deferral on the effective CGT rate. An alternative method proposed here is also valuation based, but designed for the scenario of a lump-sum realization of gradually-accrued gains, as in the case of a growth stock.

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Previous researchers recognize that the corporation's cost of retained earnings is diminished by a decrease in the effective CGT rate, but offer no guidance on how to calculate that cost under a finite deferral period. The valuation-based method provided here shows that the effect of deferral on the cost of retained earnings can be economically significant. This result suggests that deferral over a realistic finite time period may have a measurable effect on corporate investment and the choice between internal and external equity financing.

Accepted methods of calculating the effective CGT rate under deferred realization are described in Section II, followed by the derivation of our own method in Section III and comparison of the various methods in Section IV. The effect of deferred realization on the firm's cost of retained earnings is analyzed in Section V. Implications of the results for decisions by firms and individual investors, and for public policy, are discussed in Section VI.

II. Accrual-Equivalent CGT Rate as Derived by Previous Methods

A. The Traditional (TR) Method

The literature in public economics and corporate finance contains numerous statements describing the effect of deferred realization of CGT on common stocks as an interest-free loan from the government in the amount of tax deferred [8, p. 373; 16, p. 59]. If a dollar of capital which accrues at the end of year 1 is realized only at the end of year $j > 1$ (namely, deferred $j - 1$ years), at which time it is taxed at the nominal personal rate t , the accrual-equivalent effective tax rate is

$$t_e = t(1 + r)^{1-j} \quad (1)$$

where r is the relevant post-personal-tax discount rate.

B. The Mervyn King (MK) Method

King [16, p. 61] offers a variation on the traditional concept of an interest-free loan,

assuming that a fixed amount q of the remaining unrealized gain is realized in every year, starting at the date of accrual. The effective CGT rate is the value of all future tax payments on a dollar of gain accrued at the end of year 1

$$t_e = tq + \frac{tq(1 - q)}{1 + r} + \frac{tq(1 - q)^2}{(1 + r)^2} + \dots + \frac{tq(1 - q)^\infty}{(1 + r)^\infty} = t \left[\frac{q(1 + r)}{q + r} \right]. \quad (2)$$

The similarity between the TR and MK methods is in the use of the same valuation concept. The two methods convey the same numerical value of t_e under the condition $q = r / [(1 + r)^j - 1]$.

C. The Bailey-Auerbach-Protopapadakis (BAP) Method

Bailey [6], followed by Auerbach [4] and Protopapadakis [26], describe the effect of the CGT as a change in the asset's growth rate. In the absence of a CGT, a dollar investment appreciating over j years at the per-share annual rate g has a terminal value $(1 + g)^j$. If the entire gain is taxed at the rate t when realized at time j , the post-tax terminal value is $V_1 = [(1 + g)^j - 1][1 - t] + 1$. Under an accrual-based treatment, the same investment would be taxed annually at the effective rate t_e , resulting according to this method in a post-tax growth rate $g(1 - t_e)$ and a terminal value $V_2 = [1 + g(1 - t_e)]^j$. The effective CGT rate is derived by setting $V_1 = V_2$ and solving for t_e

$$t_e = \{1 + g - [(1 + g)^j - 1] \cdot [1 - t] + 1\}^{1/j} / g. \quad (3)$$

A common feature of the three methods described above is their treatment of a single accrued gain, whether realized at a point in time (TR and BAP methods) or over time (MK method). In reality, however, a gain realized at a point in time is often a sum of a series of gains accrued at various points in time. This complica-

tion is dealt with in our own valuation-based method presented next.

III. Valuation-Based (VB) Method

A. Share Valuation with Deferred Trading

Consider an all-equity corporation financing its growth entirely by retention of earnings. This simplified scenario is convenient and useful, since inclusion of debt and external equity financing would burden the exposition but not change the results.¹ Let:

P = ex-dividend share price at the beginning of year 1;

E = post-corporate-tax earnings per share accrued at the end of year 1;

b = the firm's retention ratio, the fraction of post-tax earnings retained (here also the ratio of incremental investment to earnings);

t_a = shareholders' average marginal tax rate on ordinary income, including dividends.

Already used above are the notations:

t = shareholders' average marginal statutory tax rate paid on realized capital gains;

g = expected growth rate of earnings, dividends, and price per share, determined by the firm's investment policy (see Section V);²

r = post-all-tax average marginal opportunity rate of return which can be earned by shareholders on equal-risk investment;

j = a given stock's dominant investor holding period, where $j - 1$ is the dominant CGT deferral period.

The typical investor buying a share at the price P expects to sell it for $P(1 + g)^j$, incurring a tax liability of $P[(1 + g)^j - 1]t$.³ In addition, ownership of a share of stock over j years will entail a flow of post-tax dividends growing from $E(1 - b)(1 - t_a)$ to $E(1 - b)(1 - t_a)(1 + g)^{j-1}$. This formulation differs from that of Bailey [6, pp. 26-29] and Miller [19], who apply the CGT to retained earnings rather than price

changes. The two approaches cannot show the same tax liability because they use a different tax base; in equilibrium, a dollar retained generates less than a dollar of price appreciation [23]. An implicit expression for the market price is obtained by discounting the post-tax cash flows at the post-tax discount rate, r ,

$$P = E(1 - b)(1 - t_a)[(1 + g)^0(1 + r)^{-1} + (1 + g)^1(1 + r)^{-2} + \dots + (1 + g)^{j-1}(1 + r)^{-j}] + P\{(1 + g)^j - [(1 + g)^j - 1]t\}(1 + r)^{-j}.$$

and is restated in closed form

$$P = E(1 - b)(1 - t_a)[1 - (1 + g)^j \cdot (1 + r)^{-j}](r - g)^{-1} + P\{(1 + g)^j - [(1 + g)^j - 1]t\}(1 + r)^{-j}.$$

An explicit price formula is obtained by solving this equation for P

$$P = \frac{E(1 - b)(1 - t_a)}{r - g} \cdot \left[1 + t \frac{(1 + g)^j - 1}{(1 + r)^j - (1 + g)^j} \right]^{-1}$$

for $r > g \geq 0$. (4)

As seen from (4), on the assumption of pure internal financing, the impact of the dividend tax represents a fixed fraction of the share price, depending only on the tax rate. In contrast, the impact of the CGT depends upon the post-tax discount rate, r , the per-share growth rate, g , and shareholders' average holding period, j . Under a given finite holding period, that impact is monotonically decreasing in r and increasing in g , as indicated by the role played by these variables in the expression enclosed in brackets. The relationship $r > g$ also implies a monotonic inverse effect of the holding period on the impact of the CGT for any positive growth rate. The greatest impact of this tax occurs under annual trading ($j = 1$), yielding the price

$$P = \frac{E(1-b)(1-t_d)}{r-g} \left[1 + t \frac{g}{r-g} \right]^{-1}$$

$$= \frac{E(1-b)(1-t_d)}{r-g+gt} \quad (5)$$

The impact of the tax vanishes under indefinite deferral ($j = \infty$), resulting in the share price

$$P = \frac{E(1-b)(1-t_d)}{r-g} \quad (6)$$

B. Firm-Unique Effective Tax Rate

The effect of deferred trading on the impact of the CGT can be interpreted as a change in the effective tax rate. Based on (4) and (5), the effective tax rate implied by a stock's dominant j -year holding period ($j-1$ deferral period) is derived by setting the equality

$$t_e \left[\frac{g}{r-g} \right] = t \left[\frac{(1+g)^j - 1}{(1+r)^j - (1+g)^j} \right]$$

and solving for t_e

$$t_e = t \left[\frac{r}{g} - 1 \right] \left[\frac{(1+r)^j - 1}{(1+g)^j - 1} - 1 \right]^{-1} \quad (7)$$

Consistent with the effects of g , r and j on the impact of this tax in (4), t_e is directly related to g and inversely to r and j . However, unlike the price, the effective tax rate is proportional to t . Given $g > 0$, the effective tax rate is at a maximum $t_e = t$ under annual trading ($j = 1$) and at a minimum $t_e = 0$ under indefinite deferral ($j = \infty$).⁴

C. Investor-Unique Effective Tax Rate

The individual investor holding the stock under consideration is likely to have a preferred holding period i ,⁵ a marginal statutory CGT rate t_i , and a post-tax opportunity rate of return r_i which differ from the respective parameters dominating the stock's market valuation. To de-

termine the investor-unique effect of deferral on the effective CGT rate, and with it the individual's *incentive* to defer trading, we rely on the assumption that the individual investor buys and sells the stock at prices set by the market. This implies that the stock's per-share growth rate facing all investors is the same regardless of differences in their personal tax brackets.

Consider a single i -year investment cycle over which an investor holds a share of stock. The present value of CGT paid at the statutory rate t_i on gains realized at the end of year i is $t_i[(1+g)^i - 1]/(1+r)^i$. The present value of CGT that would be paid under an alternative system of accrual taxation if gains were taxed at end of years 1, 2, ..., i at the rate t_{ie} is

$$\frac{t_{ie}g}{1+r_i} + \frac{t_{ie}g(1+g)}{(1+r_i)^2} + \dots + \frac{t_{ie}g(1+g)^{i-1}}{(1+r_i)^i}$$

$$= t_{ie}g \left[\frac{1 - (1+g)^i (1+r_i)^{-1}}{r_i - g} \right]$$

The accrual-equivalent effective tax rate is found by assuming equality of the discounted tax liabilities under the two methods of taxation, and solving for t_{ie}

$$t_{ie} = t_i \left[\frac{r_i}{g} - 1 \right] \left[\frac{(1+r_i)^i - 1}{(1+g)^i - 1} - 1 \right]^{-1} \quad (8)$$

The only difference between (7) and (8) is in the value of the parameters: firm-market parameters t , r and j , compared with parameters of the individual investor, t_i , r_i and i . The procedure leading up to (8) is mathematically equivalent to that leading up to (7). Equation (8) describes the effect of deferred realization by the single investor on the effective CGT rate paid by that investor. As such it also describes the individual investor's incentive to defer trading. The absence of the firm-unique t_e from the t_{ie} formula indicates that the effective CGT rate dominating the market for a firm's stock has no direct effect on the effective CGT rate paid by the individual. This result holds even if the market is dominated by zero-bracket investors, such as tax-sheltered retire-

ment funds. Indirectly, a lower dominant tax bracket may raise the firm's marginal efficiency of capital and with it raise g , somewhat weakening the inverse effect of i on t_e .

IV. The Four Methods Compared

A. Theory

There is no fundamental difference among the TR, MK, and VB methods—all three are valuation based. The only divergence among these methods is in the underlying scenario. The TR method assumes a lump-sum realization of a lump-sum accrual; the MK method assumes a gradual realization of a lump-sum accrual; and the VB method assumes a lump-sum realization of a gradual accrual. A fundamental difference does exist, however, between these method and the BAP method. Unlike equations (1)–(2) and (7), the investors' opportunity rate of interest does not enter equation (3); the BAP method is the only one based on the Internal Rate of Return concept rather than that of present value. The use of IRR to calculate t_e under the BAP method implies the inconsistent assumption that changes in the deferral period and statutory tax rate cause parallel changes in the opportunity rate of return. Furthermore, the distinction between pre-tax and post-tax growth rates under that method contradicts a basic corollary of market share valuation—that the tax effect is sustained by the share's rather than its growth rate, unless there is a change in the firm's financial and investment policies. This gap between the BAP method and the other three is not bridged even in the special case of $g = r$.

B. Numerical Examples

Due to their reference to different scenarios and reliance on different assumptions, not all four methods can be numerically compared. The MK method is excluded from our comparison because of the indeterminate relationship between its parameter q and parameter j used by

the other methods. The TR and BAP methods are directly comparable, but their numerical relationship depends on the arbitrary value of r absent from the BAP method. Finally, the VB method is not directly comparable with the TR and BAP methods because of the different assumptions concerning the accrual of gains. Our discussion will allow for these differences.

Figure 1 consists of three panels, one for each of the statutory CGT rates .16, .28 and .33.⁶ All three panels assume the same feasible parameters $r = .10$ and $g = .04$ or $g = .08$, allowing the holding period to change from one to twenty years.⁷ Several conclusions can be drawn from the comparisons.

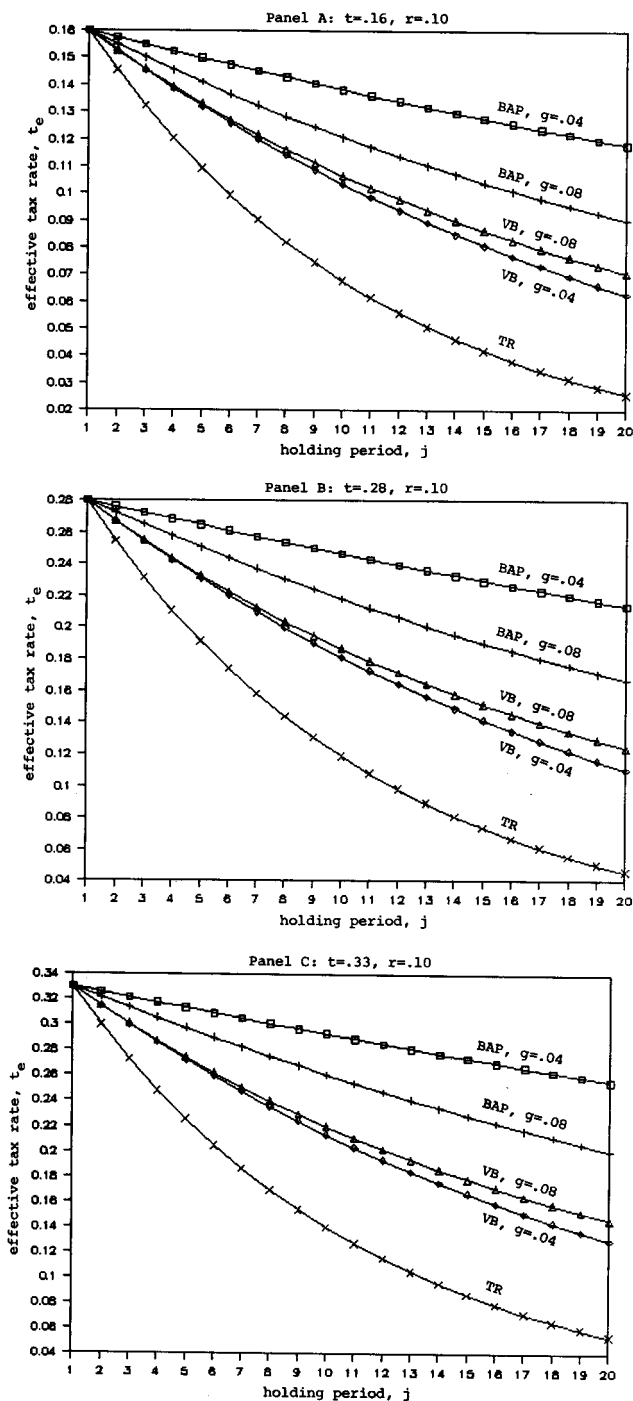
First, in reference to the TR benchmark figure, the BAP method understates the effect of deferring a lump-sum gain on the effective CGT rate. Given $t = .28$, $g = .04$ and $j = 4$ (three-year deferral), the TR method shows t_e of .2104, compared with .2683 based on the BAP method (Panel B). Under $j = 20$ (nineteen-year deferral), the difference between the two methods is greater, as reflected by the respective figures .0458 and .2138.

Second, since the TR and BAP methods are based on a lump-sum accrual, they should overstate the true effect of deferral on t_e as determined by the VB method when gain is accrued gradually. This argument is confirmed only with respect to the TR method. The bias inherent in the BAP method is so large that the calculated t_e exceeds that of the VB method under any finite deferral period. Using the same parameters as above, the VB method yields $t_e = .2425$ (above .2104 and below .2683) under $j = 4$, and $t_e = .1103$ (above .0458 and below .2138) under $j = 20$ (Panel B).

Third, the partial effect of g on t_e is positive under the VB method, but negative under the BAP method. Given $t = .28$ and $j = 10$, an increase from $g = .04$ to $g = .08$ causes a minor increase in t_e from .1811 to .1866 based on the VB method, compared with a major decrease from .2464 to .2181 based on the BAP method (Panel B).

Fourth, t_e is homogeneous in t under all but the BAP method, where a higher t causes a weaker proportional effect of j on

FIGURE 1
HOLDING PERIOD AND THE EFFECTIVE CGT RATE



t_e . Given $g = .08$, an increase from $j = 1$ to $j = 10$ causes under that method a decrease in t_e from .16 to .1207 when $t = .16$ (Panel A), compared with a parallel decrease from .33 to .2607 when $t = .33$ (Panel C). Although the absolute change is greater when t is greater, the relative change is smaller in that case. In the present example, the relative decreases are by 25 percent and 21 percent, respectively.

Fifth, although not apparent from our illustration, the partial effect of r on t_e is negative under all but the BAP method where this effect is absent.

V. Deferment and the Firm's Cost of Retained Earnings

The consequences of deferment for the impact of CGT on shareholders have further implications for the corporation's cost of retained earnings. Those implications are of immediate concern to corporate officers in charge of investment and to public policy-makers. Like the effective CGT rate, those implications must be derived from a share price valuation formula. Under the extreme assumptions of annual trading or no trading at all, the cost of retained earnings has familiar closed-form solutions which may be derived from (5) and (6), respectively. Let:

k^* = post-corporate-tax *average* rate of return earned by the firm on incremental investment, so that $g = bk^*$;

k = post-corporate-tax minimum *average* rate of return on incremental investment financed by retention, the cutoff value of k^* ; also known as the cost of retained earnings (finding this rate is the object of our analysis);

k' = post-corporate-tax *marginal* rate of return earned on investment financed by retention, defined by $k' = d(bk^*)/db$, so that $k' = k^* + b(\partial k^*/\partial b)$ at $k^* = k$.

For $j = 1$, the marginal cost of retained earnings is derived from (5) by substituting bk^* for g , and then totally differentiating the price with respect to b , setting

the derivative at zero, and solving for $k' = k^* + b(\partial k^*/\partial b)$ at $k^* = k$. In view of the possible dependence of k^* on b (in the vicinity of equilibrium, $\partial k^*/\partial b < 0$), the derivation condition takes the expanded form $dP/db = \partial P/\partial b + (\partial P/\partial k^*)(\partial k^*/\partial b) = 0$, yielding

$$k' = \frac{r}{1-t} + \frac{\partial k^*}{\partial b} b^2.$$

Under the standard simplifying assumption of $\partial k^*/\partial b = 0$ at $k^* = k$, this marginal rate is reduced to the familiar average rate (*cf.* Stiglitz [28])

$$k = k' = \frac{r}{1-t}. \quad (9)$$

For $j = \infty$, the marginal cost of retained earnings is obtained by applying a similar procedure to (6)

$$k' = r + \frac{\partial k^*}{\partial b} b^2.$$

By setting $\partial k^*/\partial b = 0$, this rate becomes the average rate

$$k = k' = r. \quad (10)$$

Previous studies do not extend the analysis beyond this point, leaving for the reader to decide how to handle the more realistic case of a finite deferral period. Based on the logic of previous methods of calculating the effect of deferral on the effective CGT rate, it is tempting to calculate k by substituting in (9) the relevant t_e . This would be incorrect. Clearly, $j = 1$ and $j = \infty$ must be viewed as special cases of the continuum $1 \leq j \leq \infty$. Under a finite deferral period, equations (4) and (7) indicate a cutoff rate k which is asymptotically decreasing with increases in the holding period, but one which has no closed-form solution. Nevertheless, that rate can be numerically evaluated for any given set of parameters r , t , b and j . Since it is apparent from (4) that P is monotonically increasing in k^* (subject to $k^* < r/b$), the critical value $k^* = k$ can be identified

by examining the effect on P of a small increment in b under increasing values of k^* , starting at $k^* = r/(1 - t)$. The cutoff rate k is signified by a change in the effect of b on P from negative to positive, since it is the minimum rate of return at which an increase in b increases P .

The partial effect of the holding period on the cost of retained earnings under alternative values of b (and g) is displayed in Figure 2, where $r = .10$ is held constant and changes in t follow the sequence of Figure 1. The following patterns emerge from the three panels.

First, the cost of retained earnings monotonically decreases by increasing the deferral period. Given $t = .28$ (see footnote 6) and $b = 0$ ($g = 0$), an increase in the holding period from $j = 1$ to $j = 10$ causes a decrease in k from .1389 to .1213 (Panel B).

Second, the effect of deferral on k is slightly influenced by the firm's retention (and growth) rate: a higher b causes a lesser effect of j on k . Given $t = .16$, an increase in the holding period from $j = 1$ to $j = 10$ causes a decrease in k from .1190 to .1112 under $b = 0$, compared with a smaller decrease from .1190 to .1120 under $b = .8$ (Panel A). Growth has a similar effect on t_e derived by the VB method (Figure 1).

Third, the cost of retained earnings is not homogeneous in the nominal tax rate: a higher t causes a greater proportional effect of j on k . Given $b = 0$, an increase in the holding period from $j = 4$ to $j = 20$ causes a decrease in k from .1160 to .1059 under $t = .16$ (Panel A), compared with a decrease from .1398 to .1131 under $t = .33$ (Panel C). The relative decreases are by 9 percent and 19 percent respectively.

VI. Implications for Private Decisions and Public Policy

The individual investor and the firm. Previous methods of computing the effect of deferred realization of capital gains on the effective tax rate paid on such gains are based on the scenario of a lump-sum accrual. The method proposed here is designed to determine the effect of deferral where gains accrue over time in a geo-

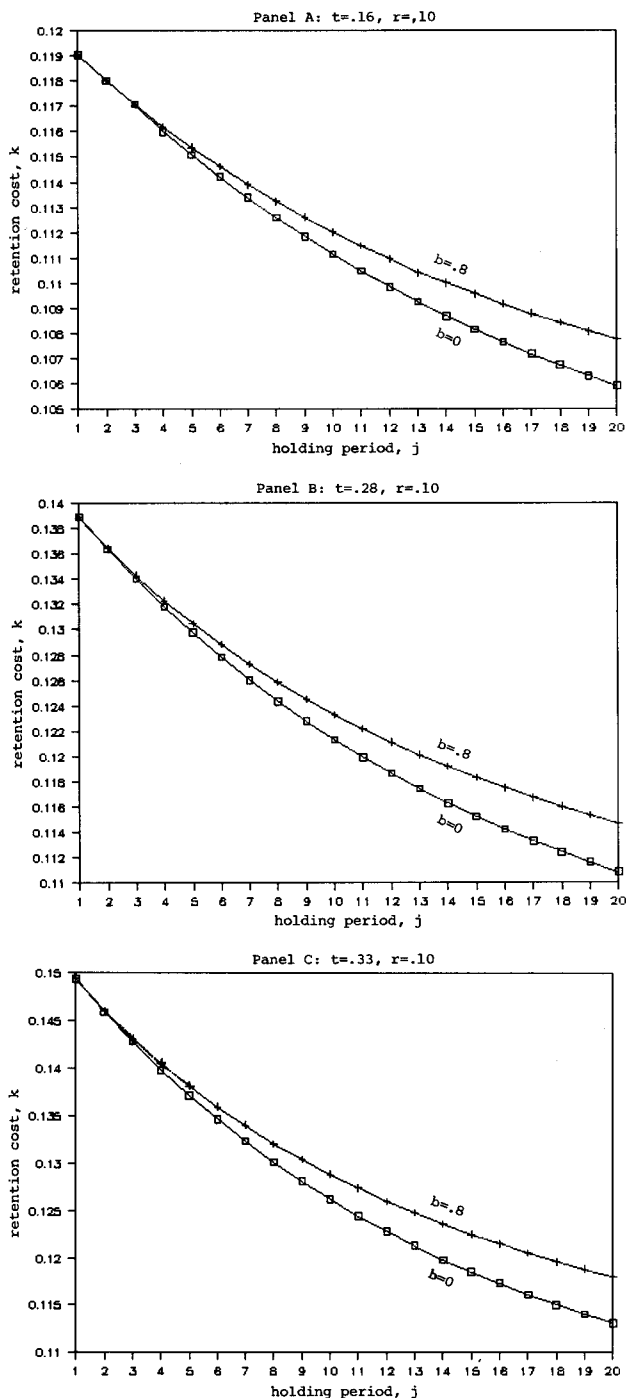
metric fashion, as in the case of a growth stock. Consistent with the TR and MK methods, but contrary to the BAP method, the new method shows that the individual's benefit from deferral is a function of the personal opportunity rate of return, as well as the statutory personal tax rate. Having used a valuation model to describe the effect of deferral on the firm's share price, we use the same model to determine the effect of deferral on the firm's cost of retained earnings. While previous studies show how a firm can compute its cost of retained earnings under extreme market conditions of no deferral or indefinite deferral, we show how this can be done under any deferral period.

Public Policy. We argue that the TR method provides the correct theoretical approach for determining the effect of deferred realization on the effective CGT rate where capital gains accrue in a lump sum. In contrast, our analysis reveals that the BAP method substantially understates that effect. Either method cannot be used in the common scenario of sustained growth, where capital gains do not accrue in a lump sum. Consistent with the TR method, the method proposed here is designed for that scenario and can be used to formulate a tax policy aimed at eliminating the incentive to defer realization. To do so, the *target effective tax rate* to be paid by the individual, t_{ie} , must be rendered invariant to the personal holding period, i . Based on (8), this could be accomplished by setting for each investor the statutory personal tax rate to be paid on realized gains at

$$t_i = t_{ie} \left[\frac{(1 + r_i)^i - 1}{(1 + g)^i - 1} - 1 \right] \left[\frac{r_i}{g} - 1 \right]^{-1}$$

where t_i may be higher than the personal ordinary tax rate(!). The firm's per-share growth rate, g , and the personal post-tax opportunity rate of return, r_i , could be approximated ex-post for any gain realized. The latter rate may be approximated based on the risk-free rate during the deferral period, adjusted for the asset's risk class and the individual's tax bracket. The idea of classifying assets into a few risk classes

FIGURE 2
HOLDING PERIOD AND THE COST OF RETAINED EARNINGS



is as practical as the present procedure of classifying assets based on rigid categories of depreciable life.

While the effective CGT rate determines the individual's incentive to hold the stock and turn it over, the cost of retained earnings determines the firm's incentive to retain and invest. Our analysis shows that, under parameters feasible in the U.S., deferred realization by shareholders may have a measurable effect on the cost of retained earnings, and therefore on corporate retention and investment. Our ability to *quantify* this effect under a finite deferral period is a step beyond previous claims that this effect vanishes under indefinite deferral, and beyond the wrong impression left by previous contributions that the extent of this effect is based on that of deferral on the effective CGT rate.

ENDNOTES

¹Stiglitz [28] first showed that under the pre-1987 partial tax exemption of capital gains, internal financing dominated external equity financing even in the absence of tax deferral. Without such an exemption, internal financing still dominates due to flotation costs.

²The assumption of constant growth is not more or less restrictive than the assumed constancy of other parameters of the model. Under the present assumption of pure internal financing of new investments, g is also the firm's growth rate.

³Allen [1], Constantinides [9, 10], and Stiglitz [29] show that by adopting appropriate portfolio strategies in a perfect capital market, investors could avoid paying capital gains tax. However, Poterba [25] demonstrates empirically that a large majority of the investing public does not engage in the alleged tax-minimizing portfolio transactions but behaves in line with the traditional view.

⁴Under the U.S. tax system, assets held until death escape the CGT but may be subject to estate taxes, unless the beneficiary is the spouse of the deceased.

⁵The empirical results of Lindsey [17] suggest that, absent tax rate changes, capital gains realizations closely follow changes in household traded wealth but are inversely related to changes in non-traded wealth.

⁶The statutory rate .16 is chosen to represent the pre-1987 period based on the most recent results available of Peterson *et al.* [24], showing for 1979 an average marginal personal tax rate of dividends of .40. The statutory inclusion factor of .4 is employed based on evidence cited by Poterba [25]. The same evidence suggests the use of an average marginal personal statutory CGT rate of .28 or .33 under the 1986 Tax Reform Act. Although relevant for measuring the typical incentive of *individuals* to defer realization of capital gains, these tax rates may over-

state those dominating the stock market. The share of tax-sheltered ownership in the stock market is difficult to gauge. At the end of 1988, the combined equity value of tax-exempt IRA equity funds amounted to 20 percent of the equity portfolio of all mutual funds [21]. Mutual funds are a major investor of tax-sheltered funds. Evidence from ex-dividend share price behavior and stock ownership clientele indicates that market pricing is dominated by positive and substantial tax rates [14].

⁷The holding period $j = 4$ is the approximate reciprocal of the average annual share turnover rate in the New York Stock Exchange for the years 1983-87 [22]. The turnover of individual stocks varies substantially, and the holding period of individual investors even more so. Furthermore, the above statistic is weighted by gross gains and losses rather than net gains, which count for tax purposes. Figures based on individual returns published by the IRS for 1962 and 1973 [30] show that in both years the average holding period of corporate stock, weighed by reported *net* capital gains, is 17 years. Using a similar approach, Protopapadakis [26] estimates an average holding period of 24-31 years.

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EFFECTIVE CAPITAL GAINS TAX RATES: A COMMENT ON COYNE, FABOZZI, AND YAARI**

DONALD W. KIEFER*

IN a recent article, Coyne, Fabozzi, and Yaari (1989, hereafter CFY) develop a "valuation-based" method of calculating the effective tax rate on capital gains. In contrasting their method with use of internal rates of return to calculate the effective tax rate, they leave the reader with several erroneous impressions which this comment attempts to correct.

CFY imply that their method provides correct results, but calculating the capital gains effective tax rate based on the internal rate of return (IRR) does not. They contrast their valuation-based (VB) method to a method used by Bailey (1969), Auerbach (1983), and Protopapadakis (1983) which they term the BAP method and which they say "is incorrectly based on the Internal Rate of Return concept." They further criticize the method because "the investors' opportunity rate of interest does not enter [the] equation" for the effective tax rate under the BAP approach.

But their development is inconsistent on this point. Their equation for the effective tax rate is developed from a discounted present value formula in which the discount rate, r , is defined as the "post-all-tax average marginal opportunity rate of return which can be earned by shareholders on equal-risk investment." The discounting equation determines the price, P , of the asset, and this equation is used to derive the formula for the effective capital gains tax rate. But if P is the present value of the asset using r as the discount rate, then r is also the after-tax internal rate of return on the asset. Hence, there is no distinction between the two methods on this issue.

The most fundamental difference between the VB method and the IRR method used by BAP is not mentioned by CFY. The two methods are based on different definitions of the capital gains effective

tax rate. The BAP method is based on the traditional definition. The effective tax rate on investment income is traditionally defined as follows:

$$t_e = \frac{r_p + r_a}{r_p} \quad (1)$$

where: t_e = the effective tax rate
 r_p = the pre-tax internal rate of return
 r_a = the after-tax internal rate of return.

The BAP method of calculating the capital gains effective tax rate can be represented by making appropriate substitutions in equation 1 as follows:

$$t_{eg} = \frac{g - r_{ag}}{g} \quad (2)$$

where: t_{eg} = the effective capital gains tax rate
 g = the growth rate of the price of the asset
 r_{ag} = the after-tax internal rate of return provided by capital gains.

The BAP method assumes that capital gain is the only source of income on the asset or, alternatively, if dividends (or other receipts) are present, they are ignored.

The VB method, in contrast, takes dividends into account. As developed by CFY, this results in a different definition for t_{eg} . The easiest way to compare the two methods is to derive an IRR-based equation from the VB equation developed by CFY.

A slightly modified version of their equation for t_{eg} (equation 7 in their article) is the following:

$$t'_{eg} = \left[\frac{r - g'}{g'} \right] \left[\frac{t_g((1 + g')^j - 1)}{(1 + r_a)^j - (1 + g')^j} \right] \quad (3)$$

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- where: t'_{eg} = the CFY effective capital gains tax rate
 g' = the expected growth rate of dividends and the share price
 t_g = the statutory capital gains tax rate
 j = the holding period.

The "''" is used here to distinguish the concepts used by CFY from those in equation 2. The term in the right brackets can be replaced with a term derived from CFY's equation 4 as follows:

$$t'_{eg} = \left[\frac{r_a - g'}{g'} \right] \left[\frac{D(1 - t_d)}{P(r_a - g')} - 1 \right] \quad 4)$$

- where: D = the dividend expected to be paid at the end of year 1; this term substitutes for the term $E(1 - b)$ in the CFY equation, where E is expected after-tax earnings and b is the firm's retention ratio
 t_d = the tax rate on dividends
 P = the share price.

Substituting r_d , the pretax rate of return provided by dividends, for D/P and simplifying yields:

$$t'_{eg} = \frac{r_d(1 - t_d) - r_a + g'}{g'} \quad 5)$$

$r_d = r_p - g'$, where r_p is the pretax internal rate of return on the asset. Substituting this relationship into equation 5 results in the following equation:

$$t'_{eg} = \frac{r_p - r_d t_d - r_a}{g'} \quad 6)$$

This is the IRR-based equation for t'_{eg} . It provides the same result as equation 7 in CFY if r in their equation is set equal to r_a in equation 6.¹ However, it is not, in general, equivalent to the traditional definition of an effective tax rate; equation 6 is equivalent to equation 2 only if $r_d = 0$.²

While equation 6 may be a useful way

of representing the capital gains effective tax rate for some purposes, potential users should be aware of its special characteristics. First, t'_{eg} changes if r_d changes, even if g' , t_g , and j do not change. This property of t'_{eg} would make it undesirable for many uses. Second, there is a stacking order issue involved in the definition of t'_{eg} . In equation 6, the dividend tax is stacked first and the capital gains tax is stacked second. That is, t'_{eg} is equal to the after-dividend-tax rate of return minus the after-tax rate of return, divided by the pretax rate of return provided by capital gains. Whether the dividend tax or the capital gains tax is stacked first in this approach is essentially arbitrary, but the calculated effective tax rates will not be the same under the different stacking orders.

CFY make two other distinctions between the methods of calculating t_{eg} . First, they claim the existing methods, including the BAP method, are "tailored to gains which accrue in a lump sum, unlike the typical scenario of gains accruing gradually over time." But this is untrue of the BAP method, as their language in describing it acknowledges. They describe the BAP formula as a measure of "a change in the asset's *growth rate*" (emphasis added) as a result of the capital gains tax. In outlining the derivation of the BAP measure, they note that in the absence of a capital gains tax, "a dollar investment *appreciating over j years at the per-share annual growth rate g* has a terminal value $(1 + g)^j$ " (emphasis added). They further state "Under an accrual-based treatment, the same investment would be taxed annually at the effective rate t_e , resulting according to this method in a *post-tax growth rate* $g(1 - t_e)$. . ." (emphasis added). Hence, contrary to their claim, the BAP method is not based on the notion of a gain that accrues as a single lump sum rather than gradually over time.

CFY also claim their VB method is fundamentally similar to the two other existing methods they summarize: the "traditional" method, which they label TR, and a method introduced by Mervyn King (1977), which they label MK. All three methods, they claim, are valuation based. They criticize the BAP method, on the

other hand, because it "implies the inconsistent assumption that changes in the deferral period and statutory tax rate cause parallel changes in the opportunity rate of return." They also criticize the implication under the BAP method that g may change in response to changes in the tax rate or the holding period. Contrary to their claim, however, these are also implications of the TR and MK methods.

Like the BAP method, the TR and MK methods of calculating the capital gains effective tax rate ignore dividends. In the absence of dividends, the relationship between r_a and g is determined by the tax rate and holding period. Without dividends the present value formula is as follows:

$$P = \frac{P\{(1 + g)^j - [(1 + g)^j - 1]t_g\}}{(1 + r_a)^j} \quad (7)$$

Simplifying this equation results in the following:

$$g = \left[\frac{(1 + r_a)^j - t_g}{1 - t_g} \right]^{1/j} - 1. \quad (8)$$

This equation makes clear that in the absence of dividends, the relationship between g and r_a is determined by t_g and j . If t_g or j change, then g and/or r_a must also. Hence, the TR and MK methods also implicitly assume that g or r_a change in response to a change in t_g or j .

The more basic difference between the VB method and the others is that the VB method is a partial equilibrium method whereas the others are general equilibrium. In the first instance, before market adjustments occur, the effect of a tax rate change may be limited to asset prices (on assets that pay dividends). Once adjustments occur, however, a new equilibrium will result. In this new equilibrium, r_a and g are likely to differ from their values before the tax change.

Hence, contrary to the implication in CFY, the internal rate of return (IRR) is not an incorrect basis for calculating the effective tax rate on capital gains. The difference between their VB method and

the BAP method is that they are based on different definitions of the effective tax rate (the BAP method is based on the traditional definition), not that the BAP method "is incorrectly based on the Internal Rate of Return concept." An IRR-based equation equivalent to the VB equation for the effective tax rate was derived, and its special characteristics were noted. Furthermore, contrary to the claim in CFY, the VB method does not differ from the BAP method with regard to the assumed pattern of capital gain accrual. A basic difference, in addition to the definitional difference, is that the VB approach is a partial equilibrium approach that focuses on changes in asset price, whereas the others discussed by CFY are general equilibrium approaches that allow changes in asset growth rates and after-tax rates of return.

ENDNOTES

**The views expressed in this paper are those of the author and do not necessarily represent the views of the Congressional Research Service or the Library of Congress.

¹Equation 6 does not impose greater informational requirements than equation 3, since r_p can be calculated using the following equation:

$$r_p = \frac{t_g[r_a - g'] \left[\frac{(1 + g')^j - 1}{(1 + r_a)^j - (1 + g')^j} \right] + r_a - g't_d}{1 - t_d}$$

It is not obvious that equation 6 provides the same result as CFY's equation 7 from their examples, but this is because their examples are incomplete. CFY illustrate the differences between the BAP and VB methods in an example assuming the opportunity rate of return is .10, the holding period is 10 years, and t_g is .28. They state that if the growth rate is .04, t'_{gg} is .1811; if the growth rate is .08, t'_{gg} is .1866. Using the BAP method, t_{gg} is .2464 if the growth rate is .04 and is .2181 if the growth rate is .08.

Their example is incomplete, however. For the assumed asset to yield the assumed opportunity rate of return, it must pay dividends. If the growth rate is .04, the dividend expected to be paid at the end of the first year must equal .06983P (these calculations assume $t_d = 0$). If the growth rate is .08, the expected dividend must equal .03493P. Assuming these dividends, equation 6 results in the same effective capital gains tax rates as the VB method.

²It may seem intuitively that equation 6 should be equivalent to equation 2. Substituting $r_p - r_d$ for g in the numerator of equation 2 and subtracting t_{gg} from t'_{gg} yields the following:

$$t'_{eg} - t_{eg} = \frac{r_d(1 - t_d) - (r_a - r_{ag})}{g}$$

At first glance, it may seem that the numerator of this equation should always equal 0, but, in general, it does not.

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EFFECTIVE CAPITAL GAINS TAX RATES: A REPLY

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WE thank Donald Kiefer for studying the technical details of our paper (*NTJ*, 1989), although we disagree with most of his criticism. Here is our point-by-point response.

a) *Capital gains in lump sum or as a series of payments.* Kiefer is correct in pointing out that our scenario of capital gains generated in a series of increments at a constant rate over a multi-year period is no different from that underlying the BAP method attributed to Bailey (1969), Auerbach (1983), and Protopapadakis (1983). This commonality only strengthens our criticism of the BAP method by allowing a direct comparison between that method and our own valuation-based (VB) method.

b) *Definition of the effective rate of capital gains tax (CGT).* Contrary to Kiefer's claim, all of the methods compared in our paper use the same definition of the effective CGT rate, t_e . It is defined as the accrual-equivalent tax rate consistent with the equation of $g^* = g(1 - t_e)$, where g is the per-share pre-CGT growth rate and rate of return from capital gains, and g^* the post-CGT growth rate and rate of return from that source.

c) *IRR vs. NPV.* Kiefer fails to see that the difference between the BAP and VB methods in calculating the effective CGT rate arises *only* where there is tax deferral. Given the same general definition of that rate, the two methods must arrive at the same calculated rate in the absence of deferral under a one-year holding period. In contrast, as reported in our Figure 1, the two methods arrive at a different effective tax rate in the presence of tax deferral, under the multi-year scenario. That difference is attributed *solely* to the use of a different time-value of money. The BAP method is incorrect in compounding CGT payments using the IRR endogenous to

each investment. In contrast, the VB method uses an exogenous discount rate commensurate with market conditions and the relevant risk of each investment. To prove that this is the only source of difference between the two methods, we show below that when properly modified, the BAP method generates the same result we derive by the VB method.

Under a regime of CGT applied at the statutory rate t to gains *realized* at the end of year j , we calculate the post-tax accumulation of a dollar investment as under the BAP method:

$$V_1 = (1 + g)^j - t [(1 + g)^j - 1].$$

Under the alternative regime of CGT applied annually to *accrued* gains at the effective rate t_e , we modify the BAP method by compounding tax payments at the post-tax opportunity interest rate, r , instead of the growth rate, g . This produces the post-tax accumulation

$$\begin{aligned} V_2 &= (1 + g)^j - t_e g [(1 + g)^0(1 + r)^{j-1} \\ &\quad + (1 + g)^1(1 + r)^{j-2} \\ &\quad + \dots + (1 + g)^{j-1}(1 + r)^0] \\ &= (1 + g)^j - t_e g \left[\frac{(1 + r)^j - (1 + g)^j}{r - g} \right]. \end{aligned}$$

The accrual-equivalent CGT rate is calculated by setting $V_1 = V_2$ and solving for t_e :

$$t_e = t \left[\frac{r}{g} - 1 \right] \left[\frac{(1 + r)^j - 1}{(1 + g)^j - 1} - 1 \right]^{-1}.$$

This is the result reported in our equation (7) (the power -1 is missing there due to a typographical error) based on the VB method.

d) *Dividends and the effective CGT rate.* Kiefer's claim that methods preceding ours of calculating the effective CGT rate ignore all sources of income but capital gains

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is factually incorrect, illogical, and irrelevant. Factually, all of the authors cited focus on the effect of CGT in common stocks in the U.S., for which cash dividends is the predominant source of value. Kiefer's disregard for the main source of net cash flows from the producing to the consuming sector is illogical. It contradicts the essence of valuation in an economy where CGT is imposed precisely on those assets which cannot be directly consumed. Finally, Kiefer's insistence on the absence of other sources of income in previous methods is irrelevant; his dividend-free formulation can be shown to accommodate dividends as a source of value and the basis of capital appreciation. His equation (7), which does not specify the source of value P , is

$$P = \frac{P[(1 + g)^j - [(1 + g)^j - 1]t]}{(1 + r)^j}$$

This price expression can be further specified by substituting for P any value, including our own dividend valuation expression (equation (4) in our paper).

Consistent with our argument that the presence of dividends as a source of value does not affect the calculation of the effective CGT rate, we proved above that our result can be obtained by modifying the BAP method without specifying the source of value.

e) *Dividend tax and the effective CGT rate.* Based on his equation (6), Kiefer claims that the effective CGT rate is a function of the firm's dividend tax rate, which renders our results incomparable with those of BAP, where no dividends are specified. To disprove this claim, we cite the above conclusion that the specific identity of the source of value has no bearing upon the effective CGT rate. Since the discounted value of dividends is homogeneous in the flat-rate dividend tax, the per-share constant growth rate, $(P_j - P_{j-1})/P_{j-1}$, which is the base for the CGT, is not affected by the dividend tax. This is based on the further assumption that in partial equilibrium dividend tax does not affect the firm's investment and financial decisions, and therefore its per-

share growth rate. With exogenous growth rate and post-tax shareholders' opportunity rate of return, our equation (7) shows an effective CGT rate that is not a function of the dividend tax.

In a related criticism, Kiefer claims that our method is unique and peculiar in defining a stacking order of the taxes on dividends and capital gains, making the latter dependent upon the former, but not the other way around. As stated, this claim is incorrect. As shown above, under all the methods compared in our paper, including our own, the effective CGT rate is independent of the dividend tax. On the other hand, under all those methods the CGT revenue partially decreases with any increase in the dividend tax that causes a decrease in the asset's value and—given the growth rate—the dollar value of the capital appreciation.

f) *What determines r and g ?* Based in the partial equilibrium surrounding the single firm, our method of deriving the effective CGT rate treats as parameters shareholders' opportunity rate of return, r , and the firm's per-share growth rate, g . Kiefer claims that such a treatment is possible only because our method is unique (his view!) in specifying dividends as the source of value. He cites his equation (8) as "proof" that in all the methods but ours, r or g must be endogenous. He goes on claiming that in the absence of dividends, g is determined by r , as well as by the statutory CGT rate, t , and the holding period, j .

This claim is incorrect. Kiefer's equation (8) is obtained by rearranging his valuation equation (7) cited above, stating g instead of P as a dependent variable. In our partial equilibrium analysis, causality goes in the following direction:

$$(r, g, t, j) \rightarrow t_e \rightarrow g^*$$

where $g^* = g(1 - t_e)$. Here r , g , t , and j are exogenous parameters that jointly determine t_e , while t_e and g jointly determine the post-tax growth rate, g^* . His equation (8), showing g as a dependent variable, merely restates equilibrium relationships, not causality.

g) *Partial vs. general equilibrium.* Kie-

fer's final claim is that the use of IRR as an opportunity rate of return for tax payments under the BAP method is correct in general equilibrium, even if our use of the cost of capital, r , is valid in partial equilibrium. This claim does not and cannot have a basis in theory, since it implies the inconsistent assumption that, in equilibrium, the CGT payments on assets of the same risk class should be discounted at interest rates which vary according to the growth rate of each asset.

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